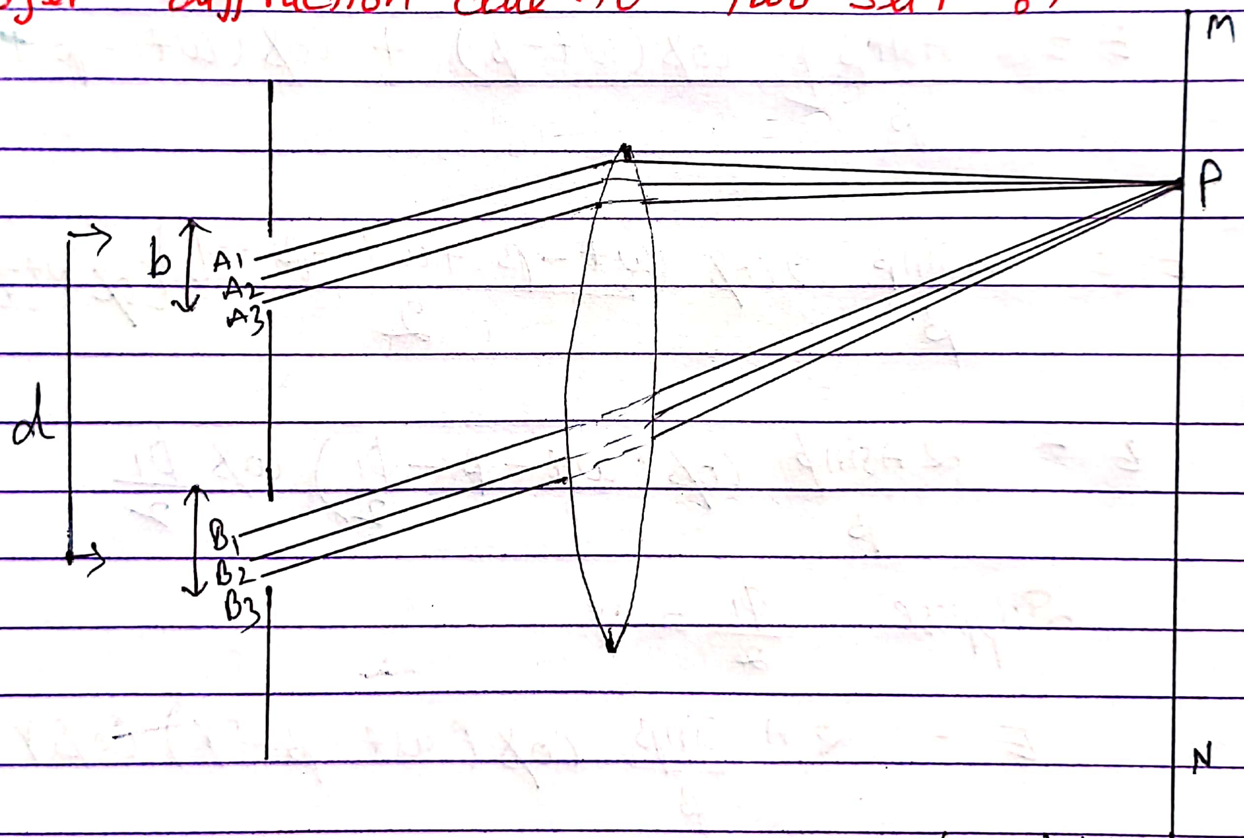
Fraunhofer diffraction due to two slit :->



we know that $E = E_0 \cos\left[\omega t - \frac{(n-1)\phi}{2}\right]$ — (1)

The electric field produced due to first slit

$$E_1 = \frac{A \sin\beta}{\beta} \cos(\omega t - \beta) \quad \text{--- (II)}$$

Similarly the field produced by 2nd slit

$$E_2 = \frac{A \sin\beta}{\beta} \cos(\omega t - \beta - \phi_1) \quad \text{--- (III)}$$

where $\phi_1 = \frac{2\pi}{\lambda} d \sin\theta$ is the phase difference between the disturbance from two corresponding points $(A_1, B_1), (A_2, B_2), (A_3, B_3)$.

where the points A_1, B_1 are separated by a distance 'd'.

Now the resultant field due to two slit

$$E = E_1 + E_2$$

$$E = \frac{A \sin\beta}{\beta} \cos(\omega t - \beta) + \frac{A \sin\beta}{\beta} \cos(\omega t - \beta - \phi_1)$$

$$E = \frac{A \sin\beta}{\beta} \left[\cos(\omega t - \beta) + \cos(\omega t - \beta - \phi_1) \right]$$

$$E = \frac{A \sin\beta}{\beta} 2 \cos\left(\frac{\omega t - \beta + \omega t - \beta - \phi_1}{2}\right) \cos\left(\frac{\omega t - \beta - \omega t + \beta + \phi_1}{2}\right)$$

$$E = \frac{2 A \sin\beta}{\beta} \cos\left(\omega t - \beta - \frac{\phi_1}{2}\right) \cos\frac{\phi_1}{2}$$

Suppose $\frac{\phi_1}{2} = \gamma$

$$E = 2A \frac{\sin\beta}{\beta} \cos(\omega t - \beta - \gamma) \cos\gamma$$

$$E = 2A \frac{\sin\beta}{\beta} \cos\gamma \cos(\omega t - \beta - \gamma) \quad \text{--- (IV)}$$

Intensity $I = (\text{Amplitude})^2$

$$I = \left(2A\frac{\sin\beta}{\beta}\cos\gamma\right)^2$$

$$I = 4A^2\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$$

$$I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma \qquad (I_0 = A^2)$$

intensity is the product of two terms

(i) $\frac{\sin^2\beta}{\beta^2}$ this is the diffraction due to single slit.

(ii) $\cos^2\gamma$ this is the interference pattern produced due to two point sources.

Minima:→ $I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$

Intensity will be minimum if

$$\beta = \pi, 2\pi, 3\pi \text{ ----}$$

or $\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$$\beta = m\pi, \quad m = 1, 2, 3 \text{ -----}$$

$$\frac{\cancel{\pi}}{\lambda} b\sin\theta = m\cancel{\pi} \qquad \left(\because \beta = \frac{\pi b\sin\theta}{\lambda}\right)$$

$$\boxed{b\sin\theta = m\lambda}$$

$$\gamma = \frac{\phi_1}{2} = \frac{\pi d\sin\theta}{\lambda} \qquad \left(\because \phi = \frac{2\pi b\sin\theta}{\lambda}\right)$$

$$\frac{\cancel{\pi} d\sin\theta}{\lambda} = \left(n+\tfrac{1}{2}\right)\cancel{\pi}$$

$$\boxed{d\sin\theta = \left(n+\tfrac{1}{2}\right)\lambda}$$

Maxima :-

$$\gamma = 0, \pi, 2\pi$$

$$\boxed{d \sin\theta = m\lambda}$$